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Shear Viscosity of Clays to Compute Viscous Resistance

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ABSTRACT: Soils at critical state are like a visco-plastic fluids, which will flow for applied stresses greater than the critical state (CS) shear strength. Shear viscosity is a key parameter to understand the post-failure flow response of soils in geotechnical applications such as landslides, jacked piles and cone penetrometers. Viscosity of soil has been investigated for water contents greater than the liquid limit because at these water contents standard viscometers can be used. However, soils can reach CS for water contents in the plastic range and thereafter exhibit viscous behavior without further changes in water contents. There is no standard method to determine shear viscosity of clays at water contents lower that the liquid limit. A new analytical method is proposed to determine the viscous drag (force) on the penetrating cone in a fall cone test. Based on this analysis, it is possible to interpret the shear viscosity of clays with water contents less than the liquid limit from a penetration test. Time-penetration data from a fall cone test was used to determine the shear viscosity of clays with liquidity index less than 1.5. The test results show that the shear viscosity decreases exponentially with an increase in liquidity index.

1 Introduction

There is certain class of problems in geotechnical engineering in which the soil is subjected to large strains (beyond failure) and the flows. Such problems include landslides and flow of soft clays around penetrating rigid bodies such as the shaft surface of a jacked pile or a sleeve of a cone penetrometer.

When a rigid shaft penetrates a fine grained soil, the soil mass at and near the tip is subjected to high stresses and fails. Soil then flows around the shaft surface during continuous penetration (Mahajan and Budhu 2006). The relationship between the resistance of the continuous penetrating rigid body and the rate of penetration depends on the stress-strain (solid) relationship and the flow (viscous) properties of the soil (Marsland and Quarterman, 1982). The total shaft resistance is due to the static shear stress (interfacial solid friction) and viscous shear stress. Studies related to the dynamic penetration of plates, probes and cone penetrometers in clays (Turnage, 1973; Murff and Coyle, 1973; Berry, 1988) show that viscous resistance is an important component of the total resistance offered by the soil.

The viscous drag force (f_z) on a rigid body moving through a viscous fluid and exhibiting low Reynolds number creeping flow is given by an equation of the following form (Lamb, 1932; Ray, 1936; Happel and Brenner, 1965; Panton, 1984)

$$f_z = C\mu_p V_z \tag{1}$$

where μ_p is the shear viscosity of the fluid, V_z is the velocity of the body and C is a constant, which is a function of the geometry of the body with respect to the flow field and the boundary conditions imposed. For an unbounded creeping flow past a sphere of radius, R, $C = 6\pi R$ (Lamb, 1932).

Mahajan and Budhu (2006) developed a similar analytical solution to determine the viscous drag on a shaft surface penetrating a clay. The solution was determined through a theoretical analysis based on viscous creeping flow of soil adjacent to the shaft. The analysis considered axial motion of a cylindrical shaft of radius, r_0, penetrating at a constant speed V_z, in a semi-infinite soil mass which has an annulus of radius, R_o, that has reached critical state (CS zone) The developed solution for the viscous drag force (f_v) per unit length of the shaft is

$$f_v = 2\pi\mu_p V_z \beta_0 \tag{2}$$

where $\beta_0 = \dfrac{\lambda_0^4 - 4\lambda_0^2 + 4ln\lambda_0 + 3}{(1 - \lambda_0^2)\left[(1 - \lambda_0^2)(ln\lambda_0 - 1) - 2ln\lambda_0\right]}$ in which $\lambda_0 = \dfrac{R_0}{r_0}$

Equation (2) reveals that the soil's viscosity, the rate of shaft penetration and the size of the critical state zone represented by β_0 or λ_0, are the key parameters controlling the viscous drag on the penetrating shaft. The term β_0 quantifies the effects of size of the CS zone on the viscous drag. The expression for viscous drag on a penetrating shaft per unit length is similar to equation (1), where $C = 2\pi\beta_o$. Shear viscosity of soil is required to determine the viscous drag on the penetrating shaft and understand post-failure behavior of soil.

In soils, the viscous drag component is often neglected because it is believed to be small in comparison with the interfacial solid friction. However, if the soil's viscosity is significant or the penetration velocity of the object is large, the viscous component (drag) could be significant. The key soil parameter for the analysis discussed in this paper is the shear viscosity for which there is no standard method for its determination except for soils at higher water contents (liquidity indices greater than 1.0). In the following, the shear viscosity of soil is discussed and the fall cone test is examined to determine its potential as a tool to estimate the shear viscosity at lower liquidity indices (LI<1.5).

2 Shear viscosity of clays for post-failure analysis

The shear viscosity (μ_p) at critical state is the desired parameter For post-failure analysis of soils,. According to Edgers and Kalrsrud (1982), the shear viscosity of soil plays an important role in landslides. Torrance (1987) stated that it is of geotechnical importance to know the flow properties (yield stress and viscosity) of soil for the purpose of understanding the post-failure behaviour.

Conventionally, viscometers have been used for measuring the yield strength (Vyalov, 1986; Torrance, 1987; Tan *et al.*, 1991; Fakher *et al.,* 1999) and shear viscosity (e.g. Locat and Demers, 1988; Komamura and Huang, 1974) of super soft clays with high water contents and very low shear strength. However, operational difficulties (Fakher *et al.*, 1999) limit the use of viscometer to clays with high liquidity index (LI >1.5). Critical state in a soil can be achieved at a liquidity index less than one. Unfortunately, the shear viscosity of soils at critical state cannot be determined using viscometers. In the following portion of this paper the fall cone (penetration) test (Hansbo, 1957; Houlsby, 1982; Koumoto and Houlsby, 2001) is explored to evaluate its potential to estimate the shear viscosity of clays at low liquidity indices. Low liquidity index in the context of this paper refers to liquidity index less than 1.5. In the next section, soil as a viscous fluid is briefly discussed to aid the interpretation of the fall cone test data.

3 Viscous fluid model for soil

When the applied stress in a visco-plastic fluid is less than a certain critical (fluid yield) stress, it deforms plastically like a solid with definite strain recovery upon the removal of stress. Applied stress exceeding the fluid yield stress will cause the fluid to flow. For simple shear flow, the constitutive equations for yield stress fluids are expressed as (Nguyen and Boger, 1992):

$$\dot{\gamma} = 0 \qquad \tau < \tau_y \,(no\ flow) \tag{3a}$$

$$\tau = \tau_y + \mu_p\dot{\gamma} \qquad \tau \geq \tau_y \,\text{(Bingham fluid)} \tag{3b}$$

where τ is the total (dynamic) shear stress, $\dot{\gamma}$ is the shear strain rate and τ_y is the yield stress. The critical state shear stress (τ_{cs}) is representative of the fluid yield stress (τ_y).

The post-failure (flow) response of clays can be represented as illustrated in Figure 1. Plastic fluids similar to curve B (Figure 1), characterized by yield stress and slowly decreasing viscosity, can be represented by the shear stress-shear strain rate relationship for plastic flow proposed by Casson (1959) as

$$(\tau)^{\frac{1}{2}} = (\tau_y)^{\frac{1}{2}} + (\mu_p\dot{\gamma})^{\frac{1}{2}}; \quad for\ \tau \geq \tau_y \tag{4}$$

This relationship (called Casson fluid) has been successfully applied to a diverse range of materials (Nguyen and Boger*,* 1992). Locat and Demers (1988) suggested that most sensitive clays behave as a Bingham or a Casson fluid and less sensitive clays can be better represented by a Casson fluid. Assuming Casson's relationship is applicable to (less sensitive) clays at low liquidity index, then in order to determine the shear viscosity, the total shear stress, the yield shear stress and the shear strain rate need to be known.
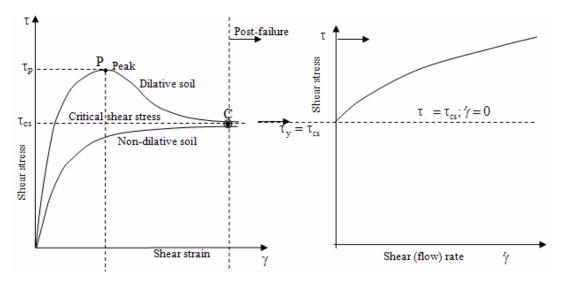
Figure 1. Analogy of the post-failure response of a soil to a yield stress fluid response

4   Shear viscosity from the fall cone test

The fall cone test is used to determine the index properties (Wroth and Wood, 1978; Wood, 1982; Budhu, 1985; Zreik *et al.*, 1995; Feng, 2000) and the undrained shear strength (Hansbo, 1957; Houlsby, 1982; Wood, 1985; Shimobe, 2000; Koumoto and Houlsby, 2001) of fine-grained soils. Hansbo (1957) showed that the undrained shear strength is related to the final depth of penetration (h_f) of the cone and can be expressed as

$$\tau_{cs} = \frac{KW}{h_f^2} \tag{5}$$

where τ_{cs} (or s_u) is the (undrained) critical state shear strength, W is the weight of the cone and K (fall cone factor) is a constant. According to Hansbo (1957), soil's resistance to the penetration of the cone depends on the static shearing resistance (interfacial friction) and viscous resistance. Theoretical and experimental work (Houlsby, 1982; Wood, 1985; Koumoto and Houlsby, 2001) showed that the cone factor (K) is influenced by the cone geometry, cone roughness, soil heave around the cone and dynamic effects.

The shear resistance (stress) of the soil on the fall cone during motion is higher due to dynamic effects. To correct for higher (dynamic) shear stress and determine shear strength of soil (equation 5), Koumoto and Houlsby (2001) proposed a reduction factor, λ, to modify the fall cone factor (K). The factor, $\lambda = 0.74$, was estimated by extending the published triaxial test results to a higher level strain rate that approximates the strain rate occurring in the fall cone test.

The fall cone test is a dynamic test. The shear resistance when the cone is in motion is a combination of the static shear resistance and the viscous shear resistance. The approach taken in this paper is to investigate the viscous shear resistance so that the shear viscosity can be estimated.When a fall cone penetrates the soil, the failed soil mass near the cone tip assumed as a yield stress fluid (Nguyen and Boger, 1992) flows around the cone as illustrated in Figure 2. The equation of motion of a cone at any penetration depth, h, in the soil is

$$ma = mg - F\tau h^2 \tag{6}$$

where m is the mass of cone, a is the acceleration of cone at depth, h, g is the acceleration due to gravity, τ is the dynamic (total) shearing resistance and F is the non-dimensional cone resistance factor (Houlsby, 82; Koumoto and Houlsby, 2001) given as

$$F = \pi N_{ch} \tan^2(\theta) \tag{7}$$

where N_{ch} is the modified bearing capacity factor of the cone, which accounts for the heave around the cone, and θ is the half cone angle. For a $30°$ (i.e. $\theta = 15°$) semi rough cone, the value of N_{ch} is 7.457 (Koumoto and Houlsby, 2001).
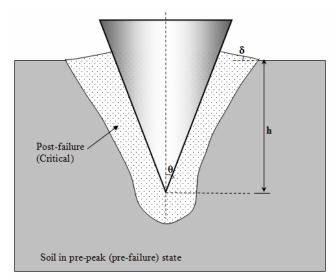
Figure 2. Illustration of the soil state around a fall cone

When the motion (free fall) of the cone is initiated, its acceleration decreases from an initial value, g, as the cone penetrates soil. At a certain depth of penetration, h_{eq}, the acceleration of the cone decreases to zero. This depth will be referred to as the dynamic equilibrium position. The cone thereafter decelerates finally coming to rest at the final depth of penetration (h_f). The dynamic equilibrium depth, h_{eq}, is less than the depth, h_s, which would be required for static equilibrium (at rest condition) of the cone. From equation (6), the dynamic equilibrium condition ($a = 0$) is

$$mg = F\tau h_{eq}^2 \tag{8}$$

from which, we get

$$\tau = \frac{mg}{Fh_{eq}^2} = \frac{W}{Fh_{eq}^2} \tag{9}$$

The shear strength of a soil using the fall cone test can be computed by using the modified cone factor, K (Koumoto and Houlsby, 2001).

Assuming the viscous soil as a Casson fluid, we can express the term $\mu_p \dot{\gamma}$ (the viscous component of shear resistance) as

$$\mu_p \dot{\gamma} = \left(\tau^{\frac{1}{2}} - \tau_{cs}^{\frac{1}{2}} \right)^2 = \left[\left(\frac{W}{Fh_{eq}^2} \right)^{\frac{1}{2}} - \left(\frac{KW}{h_f^2} \right)^{\frac{1}{2}} \right]^2 \tag{10}$$

The relationship among K, λ and F can be written as (Koumoto and Houlsby, 2001)

$$K = \frac{3\lambda}{F} \tag{11}$$

where K = 1.33 for a $30°$ semi rough cone.

Substituting equation (11) and $\lambda = 0.74$ in equation (10) and simplifying we get,

$$\mu_p \dot{\gamma} = KW \left(\frac{0.67}{h_{eq}} - \frac{1}{h_f} \right)^2 \tag{12}$$

The dynamic equilibrium depth, h_{eq}, can be obtained by measuring the cone motion (time-penetration data) and determining the penetration depth at which the acceleration is zero.

The shear strain pattern around the cone is complex and difficult to determine accurately. Consequently, the shear strain rate during the fall cone motion, which varies with the depth of penetration, is difficult to measure. However, the average shear strain rate at the equilibrium depth can be estimated from the following expression (Koumoto and Houlsby, 2001) as

$$\dot{\gamma} = \frac{2\delta}{2.44}\sqrt{\frac{g\sqrt{3}}{h_f}} \tag{13}$$

where δ the inclination angle (in radians) of the heaved soil surface (Figure 2) and is a function of the cone angle (2θ) and the cone roughness. For a $30°$ semi rough cone, $\delta = 5.77°$ (Koumoto and Houlsby, 2001). Substituting in equation (13), the average shear strain rate for a $30°$ semi-rough cone can be estimated as

$$\dot{\gamma} = 0.34\sqrt{\frac{1}{h_f}} \tag{14}$$

The shear viscosity of the soil at dynamic equilibrium from equations (12) and (14) is then

$$\mu_p = 2.94KW\sqrt{h_f}\left(\frac{0.67}{h_{eq}} - \frac{1}{h_f}\right)^2 \tag{15}$$

5 Experiment

The procedure used for conducting the tests is similar to the conventional fall cone test (Hansbo, 1957; Wroth and Wood, 1978; Wood, 1982). A standard cone of apex angle, 30° was used for the test. A calibrated LVDT was connected to the top of the cone shaft and a data acquisition system with logging frequency of 1 kHz was used to record the depth of penetration continuously with time.

Kaolin of liquid and plastic limits of 47% and 30% respectively was used for the test samples. Experiments were performed on soil samples with water contents in the range 35.8 % to 65.7 %. The total mass of the cone assembly (cone, shaft) was 93 grams. For soil states near the plastic limit, additional masses of 50 grams and 150 grams were added at the cone base to increase the depth of penetration. A typical set of experimental results, fitted with a fifth order polynomial is shown in Figure 3. The velocity and acceleration found by differentiating this polynomial are shown in Figure 4. At the equilibrium depth, h_{eq} (point A), the velocity of the cone reaches its maximum value and the acceleration is zero. The potential energy, kinetic energy and the total energy as the cone penetrates the soil are shown in Figure 5. The total energy that is present at point A causes the cone to penetrate further into the soil until all the energy is dissipated in plastic deformation of soil. Pertinent data from the experiments are summarized in Table 1.
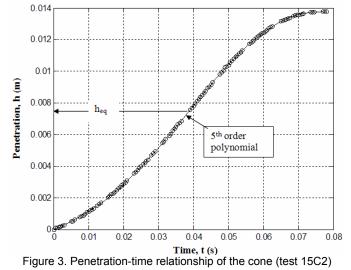
The shear viscosity (μ_p) calculated using equation (15) for different values of liquidity index (LI) together with τ_{cs} and τ are summarized in Table 1. The relationship b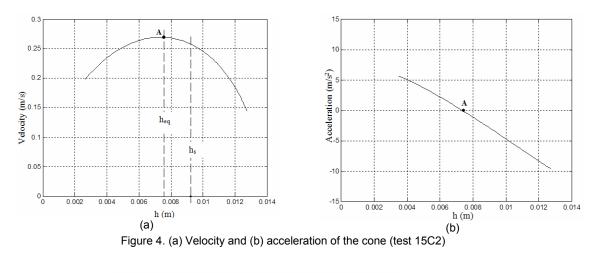etween μ_p (Pa.s) and LI for kaolin is depicted in Figure 6. The data can be fitted quite well (regression coefficient = 96%) by an exponential function expressed as

$$\mu_p = 817.6\exp(-1.43LI) \tag{16}$$

Locat and Demers (1988) proposed a similar type of relationship from viscometer data for sensitive soft clays with high LI. One of the contributing factors to the accuracy of the calculated μ_p is the accuracy of the value of K. Koumoto and Houlsby (2001) discussed some of the factors that affect the value of K and the differences between their theoretical value used here and those found by experiments that are within the range 0.7 to 1.2 for a 30° cone. The calculated μ_p is linearly related to the value of K.
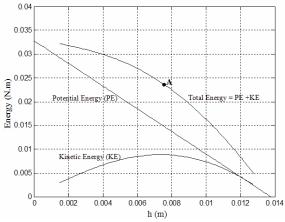
Figure 3. Penetration-time relationship of the cone (test 15C2)

(a)
(b)

Figure 4. (a) Velocity and (b) acceleration of the cone (test 15C2)

Figure 5. Potential, kinetic and total energy of the cone with penetration depth (test 15C2)

1521

Figure 6. Shear viscosity - liquidity index relationship for kaolin used in this study

$$\mu_p = 817.6 \exp(-1.43LI)$$
$$R^2 = 0.96$$

6 Conclusions

The viscous drag on a shaft penetrating in a clay depends on the size of the CS zone around the shaft, the shear viscosity of the clay in this zone and the penetration velocity of the shaft. Shear viscosity is the key soil parameter to understand post-failure viscous behavior of soil.

The shear viscosity of a soil is required to compute viscous resistance on penetrating shafts in clays. The fall cone test is shown here to be a promising tool to estimate the shear viscosity of clays at low liquidity indices. For the kaolin tested, the shear viscosity decreases exponentially with LI. However, the estimate of the shear viscosity will be dependent on the rheological model used and the value of the cone factor, K. In this paper, Casson's model was used and it is uncertain whether this model captures the viscous response of soils (in the plastic range) accurately despite suggestion in the literature of its potential appropriateness to clays.

Table 1. Test Data and Estimated shear strength and shear viscosity

Test	Water content (%)	h_f (mm)	h_{eq} (mm)	LI	τ_{cs} (kPa)	τ (kPa)	$\dot{\gamma}$ (sec^{-1})	μ (Pa.s)
15C1*	35.84	9.07	5.00	0.34	38.49	57.19	3.57	516
15C2	41.93	13.77	7.46	0.70	16.71	25.64	2.90	328
15C3	43.40	17.88	9.57	0.79	9.91	15.60	2.54	253
15C4	45.51	19.44	10.51	0.91	8.39	12.93	2.44	200
15C5	43.63	18.18	9.70	0.80	9.59	15.18	2.52	253
15C6	47.43	24.55	13.13	1.03	5.26	8.29	2.17	158
5C1**	54.28	20.40	10.88	1.43	4.48	7.10	2.38	125
5C2	54.25	23.55	12.26	1.43	3.36	5.59	2.22	127
5C3	59.89	27.59	14.64	1.76	2.45	3.92	2.05	84
C1	54.25	21.23	10.79	1.43	2.69	4.70	2.34	119
C2	65.70	30.19	16.47	2.10	1.33	2.02	1.96	36
C3	58.77	18.81	10.52	1.69	3.43	4.94	2.48	55
*Tests 15C : Cone+ additional mass of 150 gm (total mass 243 gm)								
**Tests 5C : Cone+ additional mass of 50 gm (total mass 143 gm)								
***Test C : Cone (total mass 93 gm)								

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